Applied and Computational Mathematics
2022; 11(5): 140-149
http://www.sciencepublishinggroup.com/j/acm
doi: 10.11648/j.acm.20221105.14
ISSN: 2328-5605 (Print); ISSN: 2328-5613 (Online)

Optimal Control Analysis of Meningococcal Meningitis Disease with Varying Population Size

Timothy Kiprono Yano*, Jacob Bitok, Rael Jerop

Department of Mathematics and Computer Science, University of Eldoret, Eldoret, Kenya

Email address:

yano.timothy@students.jkuat.ac.ke (Timothy Kiprono Yano), jacobbitok@yahoo.com (Jacob Bitok), raeljerop@gmail.com (Rael Jerop)

*Corresponding author

To cite this article:

Timothy Kiprono Yano, Jacob Bitok, Rael Jerop. Optimal Control Analysis of Meningococcal Meningitis Disease with Varying Population Size. *Applied and Computational Mathematics*. Vol. 11, No. 5, 2022, pp. 140-149. doi: 10.11648/j.acm.20221105.14

Received: September 20, 2022; **Accepted**: October 4, 2022; **Published**: October 17, 2022

Abstract: Meningococcal meningitis is a fatal and scary highly infectious disease especially in the African meningitis disease belt and globally, its every community's desire to wipe out meningitis disease by considering its prevention and control mechanisms. The paper formulates and analyzes a Meningococcal meningitis epidemic model that describes the spreading mechanisms of meningitis in a community with varying population. The stability analysis approach of non-linear systems is used to distinguish the properties of an epidemic deterministic compartmental model. The effective threshold reproductive value is determined by Jacobian approach and the stability study for the zero disease and endemic states are determined. Sensitivity indices analysis of the effective reproductive number to the crucial parameter values are established and rated accordingly. Using Pontryagin's approach to an optimal problem, the model was extended to include the following four control intervention measures: effort to prevent a disease infection by providing education needed, efforts to treat that minimizes sensitive and resistant strains and immunity control effort. The optimal control study of the applied control intervention efforts reveals that the use of prevention techniques and treatment efforts leads to a larger decrease of infections, thus becoming are the best intervention control strategy to eliminate the meningitis disease. Numerical analysis study was done for a combination of other strategies and main results are displayed using graphs.

Keywords: Meningococcal Meningitis, Effective Reproductive Number, Pontryagin's Principle, Optimal Intervention Strategy, Sensitivity Indices, Numerical Simulation

1. Introduction

The highly infectious meningitis disease emanates from the bacterium Neisseria meningitidis (n.m) which is a bacterial disease that is frequently occurring in the African belt. Meningococcal meningitis disease has been reported globally. Moreover, persistent recurrence of meningitis epidemics in the expansive region of sub-Saharan Africa [23], called the "meningitis belt" which consists countries ranging from western Africa to eastern Africa [12]. Meningococcal disease is life-threatening and causes high cases of child deaths in the low-income economies like Africa with an estimated 400 million cases yearly [22, 12]. Meningococci is mainly spread from one individual to another through excretions resulting from sneezing by infectious persons with asymptomatic and symptomatic stages of meningococcal disease [10]. The symptoms visible in bacterial meningitis infected individual include: sudden onset of fever, severe headache accompanied by a stiff neck, nausea, vomiting, eye sensitivity to light (photophobia) etc. [11, 14]. The discovery of vaccines (MACV) introduced to the disease affected regions in Africa has resulted to a remarkable reduced prevalence of n.m in the region with high possibility of eradication of Neisseria meningitidis. Health agencies have projected that with a high vaccination coverage rate for persons with ages 1-29 years (i.e., an estimated 315 million young people), then meningococcal disease will be eradicated from meningitis belt region of Africa [12, 14].

Over the years, the development of epidemic models has been used to explain transmission growth of the infectious meningitis disease. Notably, mathematical modelling has been significant guide to development and implementation of policy measures undertaken by health agencies to curb the

transmission of infectious diseases, just to mention a few studies by [1, 3, 4, 8-10, 15, 19] which attempt to explain the transmission modes of the n.m infectious disease agent. Similarly, a number of research findings on optimal control analysis of intervention strategies applied to n.m spreading ways and considering the effect on disease transmission has been undertaken by some researchers, although different studies had unique focus and targets such as studies by [2, 17, 20, 22]. For instance, a study by Blyuss formulated and did extensive work on a mathematical model that attempted to describe the transmission and intervention efforts of Meningitis infection [4]. Vareen [19] assessed the impact of vaccination program on meningitis transmission in the community. Clearly, a few studies have assessed the dynamics of meningitis using a varying population size. In addition to a varying population size, this study introduces a drug resistant strain of meningitis together with a drug sensitive strain and undertake a study on effective intervention control efforts.

2. Meningitis Model Formulation

The model has 5 compartments as follows: A susceptible, S, drug sensitive infectives, I, drug resistant infectives, R_T and recovery group, R. The population proportion, p was estimated to have gotten the vaccine before entry to the population and take $(1-p)$ as susceptible to the infection with recruitment rate, φ and waning rate, π. The susceptible individual gets infection through contact with drug sensitive infective or through contact with drug resistant infectives with infection rate of $\chi = \xi\left(\frac{I(t)+\Upsilon R_T(t)}{N}\right)$, where $\xi = \kappa\tau$ is the effective contact rate, κ is the rate of getting infected, τ is the possibility of an interaction to be effective in spreading infection and Υ is estimated spread coefficient for the drug resistant. μ being deaths occurring due to natural circumstances, α_1 represents deaths due to a drug sensitive disease infection while α_2 is the deaths caused by infection from drug resistant group. δ is the progression rate from infective but sensitive to treatment group to drug resistant group, θ is the progression rate from drug resistant group to recovered group and ω is the progression rate from sensitive infectives to recovered group due to treatment. u_1 represents a prevention control measure, that protects individuals from catching the disease. u_2 are efforts to treat infected persons, that minimizes infection by treating drug sensitive infective individuals. u_3 represents treatment control attempt, that minimizes infections by treating the drug resitant individuals. u_4 is an immunity effort, that signifies a decrease in waning rate as a result of high vaccination implemented and efficacy of vaccines. The resulting meningitis model nonlinear system is given by;

$$\begin{cases} \frac{dS}{dt} = (1-p)\varphi N + (1-u_4)\pi R - (1-u_1)\chi S - \mu S \\ \frac{dI}{dt} = (1-u_1)\chi\rho S - u_2\omega I - (\delta+\mu+\alpha_1)I \\ \frac{dR_T}{dt} = (1-u_1)\chi(1-\rho)S + \delta I - u_3\theta R_T - (\mu+\alpha_2)R_T \\ \frac{dR}{dt} = q\varphi N + u_2\omega I + u_3\theta R_T - (1-u_4)\pi R - \mu R \end{cases} \quad (1)$$

with initial conditions $S(0) = S_0, I(0) = I_0, R_T(0) = R_{T0}$ and $R(0) = R_0$ with $N = S + I + R_T + R$. A sum of the model equations in (1) gives,

$$\frac{dN}{dt} = (\varphi - \mu)N - \alpha_1 I - \alpha_2 R_T \quad (2)$$

We normalize the variables, $s = \frac{S}{N}, i = \frac{I}{N}, r_T = \frac{R_T}{N}$ and $r = \frac{R}{N}$ such that, the new system gives;

$$\begin{cases} \frac{ds}{dt} = (1-p)\varphi + (1-u_4)\pi r - \big((1-u_1)\xi(i+r_T)+\varphi\big)s + \alpha_1 si + \alpha_2 s r_T \\ \frac{di}{dt} = (1-u_1)\rho\xi(i+\Upsilon r_T)s - (u_2\omega+\delta+\varphi+\alpha_1)i + \alpha_1 i^2 + \alpha_2 i r_T \\ \frac{dr_t}{dt} = (1-u_1)(1-\rho)\xi(i+\Upsilon r_T)s + \delta i - (u_3\theta+\varphi+\alpha_2)r_T + \alpha_1 i r_T + \alpha_2 r_T^2 \\ \frac{dr}{dt} = q\varphi + u_2\omega i + u_3\theta r_T - \big((1-u_4)\pi+\varphi\big)r + \alpha_1 i r + \alpha_2 r_T r \end{cases} \quad (3)$$

The system (3) can be reduced by setting $r = 1 - s - i - r_T$, which yileds a subsystem,

$$\begin{cases} \frac{ds}{dt} = (1-p)\varphi + (1-u_4)\pi(1-s-i-r_T) - \big((1-u_1)(\xi(i+\Upsilon r_T)+\varphi)s + \alpha_1 si + \alpha_2 s r_T \\ \frac{di}{dt} = (1-u_1)\rho\xi(i+\Upsilon r_T)s - (u_2\omega+\delta+\varphi+\alpha_1)i + \alpha_1 i^2 + \alpha_2 i r_T \\ \frac{dr_t}{dt} = (1-u_1)(1-\rho)\xi(i+\Upsilon r_T)s + \delta i - (u_3\theta+\varphi+\alpha_2)r_T + \alpha_1 i r_T + \alpha_2 r_T^2 \end{cases} \quad (4)$$

3. Model Properties

The feasible region of model variables in (4) in $\mathbb{R}_+^3$ are confined in;

$$\Omega = \{(s, i, r_T) \in \mathbb{R}_+^3 : 0 \le s + i + r_T + r \le 1\} \quad (5)$$

so that the Meningitis model is epidemiologically and mathematically well posed.

The model was analyzed qualitatively in the set Ω. In the absence of Meningitis infection, system gives a disease-free state, $DFE = (s_0, i_0, r_{T0}) = \left(\frac{(1-p)\varphi+\pi}{\varphi+\pi}, 0, 0\right)$. The Jacobian, J_{DFE} of model equations in (4) at DFE;

$$J_{DFE} = \begin{bmatrix} -(\varphi+\pi) & -\pi - \xi s_0 + \alpha_1 s_0 & -\pi - \xi \Upsilon s_0 + \alpha_2 s_0 \\ 0 & p\xi s_0 - (\omega + \delta + \varphi + \alpha_1) & p\xi \Upsilon s_0 \\ 0 & (1-p)\xi s_0 + \delta & (1-p)\xi \Upsilon s_0 - (\theta + \varphi + \alpha_2) \end{bmatrix} \quad (6)$$

We compute $|J_{DFE} - \lambda I| = 0$, by applying the Jacobian matrix approach gives the effective reproduction number, R_e:

$$R_e = \frac{\rho\xi((1-p)\varphi+\pi)}{(\varphi+\pi)(\omega+\delta+\varphi+\alpha_1)} + \frac{\xi\Upsilon(1-\rho)((1-p)\varphi+\pi)}{(\varphi+\pi)(\theta+\varphi+\alpha_2)} + \frac{\delta\rho\xi\Upsilon((1-p)\varphi+\pi)}{(\varphi+\pi)(\omega+\delta+\varphi+\alpha_1)(\theta+\varphi+\alpha_2)} \quad (7)$$

By applying the findings in [20], the result was found to be.

Theorem 1. The DFE of the model (4), given by R_e, is locally asymptotically stable if $R_e < 1$, and unstable if $R_e > 1$.

The Global stability study reveals that the maximum invariant set contained in the set $\{(s, i, r_T) \in \Omega: \frac{dV}{dt} = 0\}$, with the Lyapunov function $V = (\theta + \varphi + \alpha_2)i + (\omega + \delta + \varphi + \alpha_1)\Upsilon r_T$, is the disease-free equilibrium. Global asymptotic stability for DFE was determined by using Lasalle-Lyapunov theorem $R_e < 1$.

4. Sensitivity Analysis of Model Parameters

We determine the sensitive parameters of the model by computing their indices that will be helpful in determining their impact on the model transmission dynamics [5].

Definition. In [5] the normalized forward sensitivity index of a parameter, R_e, with respect to a parameter, ξ, is given by, $Z_\xi^{R_e} = \frac{\partial R_e}{\partial \xi} * \frac{\xi}{R_e}$. The sensitivity indices of R_e were determined using the parameter values available in Table 2.

Table 1. *Analysis of sensitivity values of* R_e.

Variables	Variable description	Sensitivity indices
ξ	Infection rate	1.006
p	vaccinated proportion	−0.9410
Υ	transmission coefficient	0.6346
φl	recruitment rate	−0.4072
θ	progression rate from resistant to recovered	−0.28201
π	loss of immunity	0.2702
ω	progression rate from sensitive to recovered	−0.2701
ρ	Sensitive infective proportion	−0.2197
α_2	death rate due to resistant strain	−0.2115
α_1	death rate due to sensitive strain	−0.1367
δ	progression rate from sensitive to resistant strain	0.04394

Table 1 above displays the computed sensitivity values of R_e for formulated meningitis model. The indices have been aligned from the highest values to the lowest values in terms of their sensitivity index. The highest impact of all the parameters was found to be from the infection rate ξ, closely followed by vaccinated proportion at birth, p and then resistant strain transmission rate (Υ) and recruitment rate (φ) follow, while the lowest impact was witnessed in the progression rate from sensitive to resistant strain, δ. From Table 1, if the parameters ξ, Υ, π and δ are increased while the rest are held steady, the computational value of R_e rises. This implies, the parameter's increase the persistence of the meningitis disease in the community since they possess positive sensitivity indices. Also, the parameters $p, \varphi, \theta, \omega, \rho, \alpha_2$ and α_1 decreases the numerical value of R_e when it rises while holding steady the rest of the values. Thus, it reduces the persistence of the infection due to presence of a negative index.

5. Optimal Control of the Meningitis Model

The optimal intervention techniques of the system (1) are incorporated. Its significance is to identify the better control approaches which may contribute more to the elimination the meningitis infection in the community.

On incorporating time dependent control parameters, u_1, u_2, u_3 and u_4 in meningitis model, giving the optimal control model problem of meningitis described by system (3).

Epidemiologically, the idea of optimality analysis endeavors to reduce the spread of an infectious disease, minimize replication of infections and decrease costs incurred while treating and providing prevention measures as elaborated in Pontryagin [16], Fleming & Rishel [6]. To achieve the optimal control levels, define a control set U that is Lebesgue measurable as: $= \{(u_1(t); u_2(t); u_3(t); u_4(t)): 0 \le u_1 < 1; 0 \le u_2 < 1; 0 \le u_3 < 1; 0 \le u_4 < 1; 0 \le t \le T\}$. Optimal objective function J is defined as:

$$J = \min_{u_1,u_2,u_3,u_4} \int_0^{t_f} (w_1 i + w_2 r_T + w_3 u_1^2 + w_4 u_2^2 + w_5 u_3^2 + w_6 u_4^2) \quad (8)$$

With respect to the formulated equations in (1) with w_1, w_2, w_3, w_4, w_5 and w_6 representing the positive weight values to help to moderate the variables. They serve as the balancing cost factors in the optimal model.

$w_1 i, w_2 r_T$; represent the costs associated with infective human population.

$w_3 u_1^2$; represents the cost associated with prevention strategies of the susceptible population (s).

$w_4 u_2^2$; represents the costs incurred when treating drug senitive infectious individuals (i).

$w_5 u_3^2$; represents the costs incurred when treating drug resistant infectious individuals (r_T). The costs emanating

from in the treatment of the infected humans who have developed an antibiotic resistant bacterium.

$w_6u_4^2$; represents the costs incurred in immunity control to prevent loss of immunity by recovered individuals (r).

t_f; This is the duration that an intervention strategy has taken.

$w_1i + w_2r_T$; is linear and represents the cost incurred due to transmission of an infection.

$w_3u_1^2 + w_4u_2^2 + w_5u_3^2 + w_6u_4^2$; is a quadratic form function that represents costs incurred due to provision of control intervention techniques [7].

The model controls are bi-linear combination of $u_i^2(t), (i = 1,2,3)$. The quadratic form is applied because in nature costs are not linear. The goal is to decrease the replicative dynamics of infections (drug sensitive and resistant) and treating cost of the disease.

We seek to determine the optimal functions;

$$(u_1^*(t), u_2^*(t), u_3^*(t), u_4^*(t)): J(u_1^*, u_2^*, u_3^*, u_4^*) = \min\left\{\frac{J(u_1,u_2,u_3,u_4)}{u_i} \in U\right\} \tag{9}$$

where $U = \{(u_1, u_2, u_3, u_4): 0 \le u_i < 1; i = 1,2,3,4; 0 \le t \le t_f\}$ is the control set of the model.

The Lagrangian function;

$$L(i; r_T; u_1; u_2; u_3; u_4; t) = w_1i + w_2r_T + w_3u_1^2 + w_4u_2^2 + w_5u_3^2 + w_6u_4^2 \tag{10}$$

The Pontryagin's approach sets out the needed and sufficient thresholds to be satisfied by an optimal problem [16, 6]. The principle changes the system of differential equations in (1) and system (8) into a Hamiltonian (H), with controls $(u_1; u_2; u_3; u_4)$.

$$H(s, i, r_T, r, t) = L(i; r_T; u_1; u_2; u_3; u_4; t) + \lambda_1\frac{ds}{dt} + \lambda_2\frac{di}{dt} + \lambda_3\frac{dr_T}{dt} + \lambda_4\frac{dr}{dt} \tag{11}$$

Substituting the equations in system (1) the Hamiltonian becomes:

$$\begin{cases} H = w_1i + w_2r_T + w_3u_1^2 + w_4u_2^2 + w_5u_3^2 + w_6u_4^2 \\ +\lambda_1\{(1-p)\varphi + (1-u_4)\pi r - ((1-u_1)\xi(i + rr_T) + \varphi)s + \alpha_1si + \alpha_2sr_T\} \\ +\lambda_2\{(1-u_1)\rho\xi(i + rr_T)s - (u_2\omega + \delta + \varphi + \alpha_1)i + \alpha_1i^2 + \alpha_2ir_T\} \\ +\lambda_3\{(1-u_1)(1-\rho)\xi(i + \Upsilon r_T)s + \delta i - (u_3\theta + \varphi + \alpha_2)r_T + \alpha_1ir_T + \alpha_2r_T^2\} \\ +\lambda_4\{q\varphi + u_2\omega i + u_3\theta r_T - ((1-u_4)\pi + \varphi)r + \alpha_1ir + \alpha_2r_Tr\} \end{cases} \tag{12}$$

with $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ being the the adjoint variables.

Using the relation;

$$\frac{d\lambda_i}{dt} = -\frac{\partial H}{\partial\dot{x}(t)} \tag{13}$$

Differentiation of the Hamiltonian, H gives;

$$\begin{aligned} \frac{d\lambda_1}{dt} = -\frac{\partial H}{\partial s(t)} &= -\begin{Bmatrix} -\lambda_1(1-u_1)\xi(i + \Upsilon r_T) - \lambda_1\varphi + \\ \lambda_1(\alpha_1i + \alpha_2r_T) + \lambda_2(1-u_1)\rho\xi(i + \Upsilon r_T) \end{Bmatrix} \\ &= (1-u_1)\xi(i + \Upsilon r_T)(\lambda_1 - \rho\lambda_2) + \lambda_1(\varphi - \alpha_1i - \alpha_2r_T) \end{aligned} \tag{14}$$

Computing $-\frac{\partial H}{\partial\dot{x}(t)}$ of the Hamiltonian, H which gives the adjoint variables stated below;

$$\begin{cases} \frac{d\lambda_1}{dt} = (1-u_1)\xi(i + \Upsilon r_T)(\lambda_1 - \rho\lambda_2) + \lambda_1(\varphi - \alpha_1i - \alpha_2r_T) \\ \frac{d\lambda_2}{dt} = -w_1 + (1-u_1)\xi s\lambda - \rho\lambda_2 - (1-\rho)\lambda_3 - \alpha_1x + \lambda_2m - \lambda_3\delta - \lambda_4u_2\omega \\ \frac{d\lambda_3}{dt} = -w_2 + (1-u_1)\xi\Upsilon s\lambda - \alpha_2y + \lambda_3n - \lambda_4u_3\theta \\ \frac{d\lambda_4}{dt} = (1-u_4)\pi(\lambda_4 - \lambda_1) + \lambda_4(\varphi - \alpha_1i - \alpha_2r_T) \end{cases} \tag{15}$$

where $\lambda = \lambda_1 - \rho\lambda_2 - (1-\rho)\lambda_3, x = \lambda_1s + 2\lambda_2i + \lambda_3r_T + \lambda_4r, y = \lambda_1s + \lambda_2i + 2\lambda_3r_T + \lambda_4r, m = u_2\omega + \delta + \varphi + \alpha_1 - \alpha_2r_T$ and $n = u_3\theta + \varphi + \alpha_2 - \alpha_1i$

such that $\lambda_i(t_f) = 0;$ with $i = 1,2,3,4.$

Combining the Pontryagin's approach of the optimal problem as stated by Pontryagin [16] and characterizing the optimal model by solving;

$$\frac{\partial H}{\partial u_i} = 0, \text{with } u_i = u_i^*, i = 1,2,3, \dots, n. \tag{16}$$

we obtain the control set $(u_1^*; u_2^*; u_3^*; u_4^*)$ by using equation (16) as follows

$$\frac{\partial H}{\partial u_1} = 2w_3u_1 + \lambda_1\xi(i + \Upsilon r_T)s - \lambda_2\rho\xi(i + \Upsilon r_T)s - \lambda_3(1-\rho)\xi(i + \Upsilon r_T)s$$

Thus,

$$u_1^* = \frac{(\lambda_3(1-\rho)+\rho\lambda_2-\lambda_1)\xi\left(i+\Upsilon_{r_T})s\right.}{2w_3}$$

We differentiate the Hamiltonian function obtained and applying equation (16), to get the control set $(u_1^*; u_2^*; u_3^*)$ as follows;

$$\begin{cases} u_1^* = \frac{((1-\rho)\lambda_3+\rho\lambda_2-\lambda_1)\xi\left(i+\Upsilon_{r_T}\right)s^*}{2w_3} \\ u_2^* = \frac{(\lambda_2-\lambda_4)\omega i^*}{2w_4} \\ u_3^* = \frac{(\lambda_3-\lambda_4)\theta r_T^*}{2w_5} \\ u_4^* = \frac{(\lambda_1-\lambda_4)\pi r^*}{2w_6} \end{cases} \tag{17}$$

Theorem 2. The optimal control vector $(u_1^*(t), u_2^*(t), u_3^*(t), u_4^*(t))$ that maximize the objective functional, J over control set U, given by;

$$\begin{cases} u_1^*(t) = max\left\{0, \min\left(1, \frac{((1-\rho)\lambda_3+\rho\lambda_2-\lambda_1)\xi\left(i+r_{T_T}\right)s^*}{2w_3}\right)\right\} \\ u_2^*(t) = \max\left\{0, \min\left(1, \frac{(\lambda_2-\lambda_4)\omega i^*}{2w_4}\right)\right\} \\ u_3^*(t) = \max\left\{0, \min\left(1, \frac{(\lambda_3-\lambda_4)\theta r_T^*}{2w_5}\right)\right\} \\ u_4^*(t) = \max\left\{0, \min\left(1, \frac{(\lambda_1-\lambda_4)\pi r^*}{2w_6}\right)\right\} \end{cases} \tag{18}$$

where $\lambda_1, \lambda_2, \lambda_3, \lambda_4$ are the solutions of equation (12) and (15).

Writing in terms of bounds of the control variables gives;

$$u_1^* = \begin{cases} 0 & \text{if } \Theta_1 \le 0, \\ \Theta_1 & \text{if } 0 < \Theta_1 < 1, \\ 1 & \text{if } \Theta_1 \ge 1. \end{cases}$$

$$u_2^* = \begin{cases} 0 & \text{if } \Theta_2 \le 0, \\ \Theta_2 & \text{if } 0 < \Theta_2 < 1, \\ 1 & \text{if } \Theta_2 \ge 1. \end{cases}$$

$$u_3^* = \begin{cases} 0 & \text{if } \Theta_3 \le 0, \\ \Theta_3 & \text{if } 0 < \Theta_3 < 1, \\ 1 & \text{if } \Theta_3 \ge 1. \end{cases}$$

and

$$u_4^* = \begin{cases} 0 & \text{if } \Theta_4 \le 0, \\ \Theta_4 & \text{if } 0 < \Theta_4 < 1, \\ 1 & \text{if } \Theta_4 \ge 1. \end{cases}$$

with,

$$\Theta_1 = \frac{\left((1-\rho)\lambda_3+\rho\lambda_2-\lambda_1\right)\xi\left(i+Y_{r_T}\right)s^*}{2w_3},$$

$$\Theta_2 = \frac{(\lambda_2-\lambda_4)w^*}{2w_4}, \Theta_3 = \frac{(\lambda_3-\lambda_4)\theta r_T^*}{2w_5} \text{ and } \Theta_4 = \frac{(\lambda_1-\lambda_4)\pi r^*}{2w_6}$$

The resulting optimality system to be simulated is given as follows:

$$\begin{cases} \frac{ds}{dt} = (1-p)\varphi + (1-u_4^*)\pi r - \big((1-u_1^*)\xi(i+\Upsilon_T) + \varphi\big)s + \alpha_1 si + \alpha_2 s r_T \\ \frac{di}{dt} = (1-u_1^*)\rho\xi(i+r_T)s - (u_2^*\omega + \delta + \varphi + \alpha_1)i + \alpha_1 i^2 + \alpha_2 i r_T \\ \frac{drt}{dt} = (1-u_1^*)(1-\rho)\xi(i+r_T)s + \delta i - (u_3^*\theta + \varphi + \alpha_2)r_T + \alpha_1 i_T + \alpha_2 r_T^2 \\ \frac{dr}{dt} = q\varphi + u_2^*\omega i + u_3^*\theta r_T - \big((1-u_4^*)\pi + \varphi\big)r + \alpha_1 i r + \alpha_2 r_T r \\ \frac{d\lambda_1}{dt} = (1-u_1^*)\xi(i+r_T)(\lambda_1 - \rho\lambda_2) + \lambda_1(\varphi - \alpha_1 i - \alpha_2 r_T) \\ \frac{d\lambda_2}{dt} = -w_1 + (1-u_1^*)\xi s(\lambda_1 - \rho\lambda_2 - (1-\rho)\lambda_3) - \alpha_1 x + \lambda_2 m - \lambda_3\delta - \lambda_4 u_2^*\omega \\ \frac{d\lambda_3}{dt} = -w_2 + (1-u_1^*)\xi\Upsilon s(\lambda_1 - \lambda_2\rho - \lambda_3(1-\rho)) - \alpha_2 y + \lambda_3 n - \lambda_4 u_3^*\theta \\ \frac{d\lambda_4}{dt} = (1-u_4^*)\pi(\lambda_4 - \lambda_1) + \lambda_4(\varphi - \alpha_1 i - \alpha_2 r_T) \end{cases} \quad (19)$$

with conditions $\lambda_i(t_f) = 0$, where $i = 1,2,3,4$. and initials being $s(0) = s_0, i(0) = i_0, r_T(0) = r_{T0}$ and $r(0) = r_0$.

6. Numerical Simulations

Numerical solution of the model equations in (1) are done together with the optimality system by employing given values in Table 2 to perform the analysis.

The following estimates were used; $w_1 = 300; w_2 = 150; w_3 = 2; w_4 = 2; w_5 = 4$ and $w_6 = 6$ for estimation of the solution of a meningitis epidemic with optimality of the problem considered. Additionally, the initial conditions $s(0) = 0.4; i(0) = 0.3; r_T(0) = 0.2; r(0) = 0.1$and final conditions $\lambda_1(t_f) = 0, \lambda_2(t_f) = 0, \lambda_3(t_f) = 0$ and $\lambda_4(t_f) = 0$ were used.

Table 2. *Table showing values used to analyze the meningitis equations.*

Parameters	ξ	p_c	δ	ω	φ	θ	α_1	α_2	Υ	π	ρ
values	0.89	0.6806	0.15	0.3	0.1	0.2	0.2	0.15	1.2	0.04	0.6
Ref.	[19, 1]	assumed	assumed	[19]	[21, 1]	[3]	[13, 4]	assumed	[18]	[9, 2]	fitted

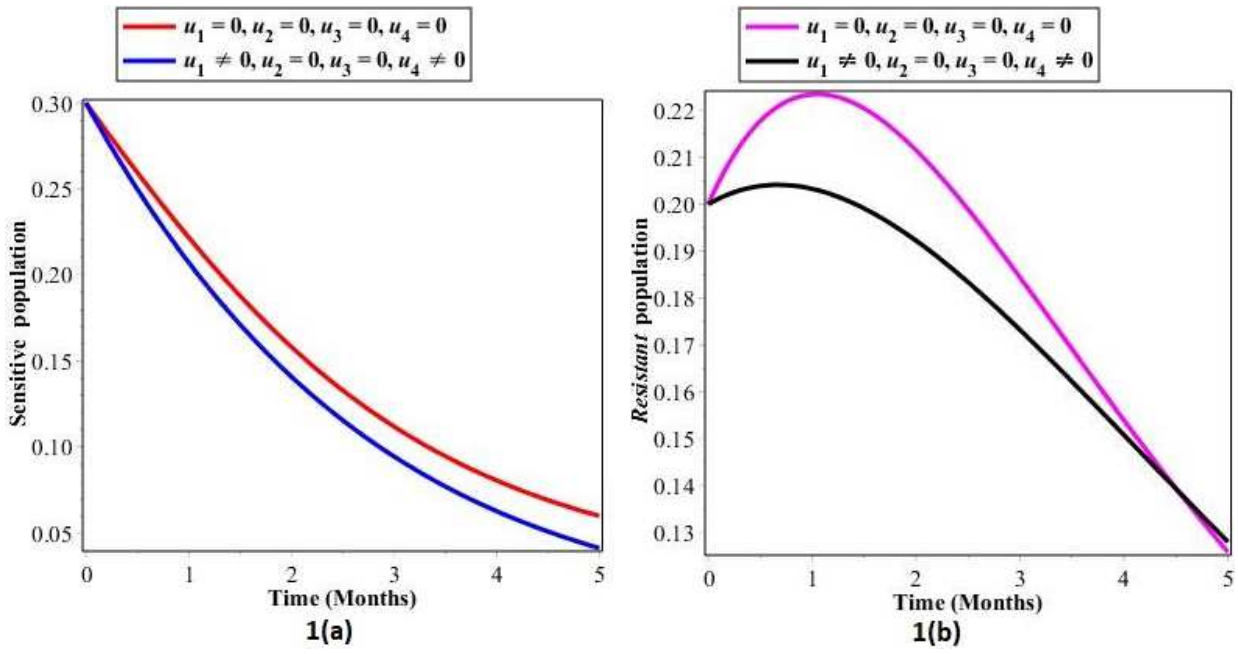

Figure 1. *Graph displaying the impact of intervention strategies:* $u_1 \neq 0$ *and* $u_4 \neq 0$.

Figure 1(a) and 1(b) illustrates the case when application of the control efforts u_1 and u_4 are non-zero. The controls were significant in maximizing the function J while equating the controls u_2 and u_3 to zero. Clearly, Figures 1(a) shows the technique has less impact on decreasing the population of drug sensitive individuals. On the other hand, Figures 1(b) displays a significant notable effect of the controls on timing and attainment of peak infections on drug resistant individuals (r_T), this may be as a result of the education campaigns on effective use and administration of drugs as per physicians' guidance on infective but resistant individuals.

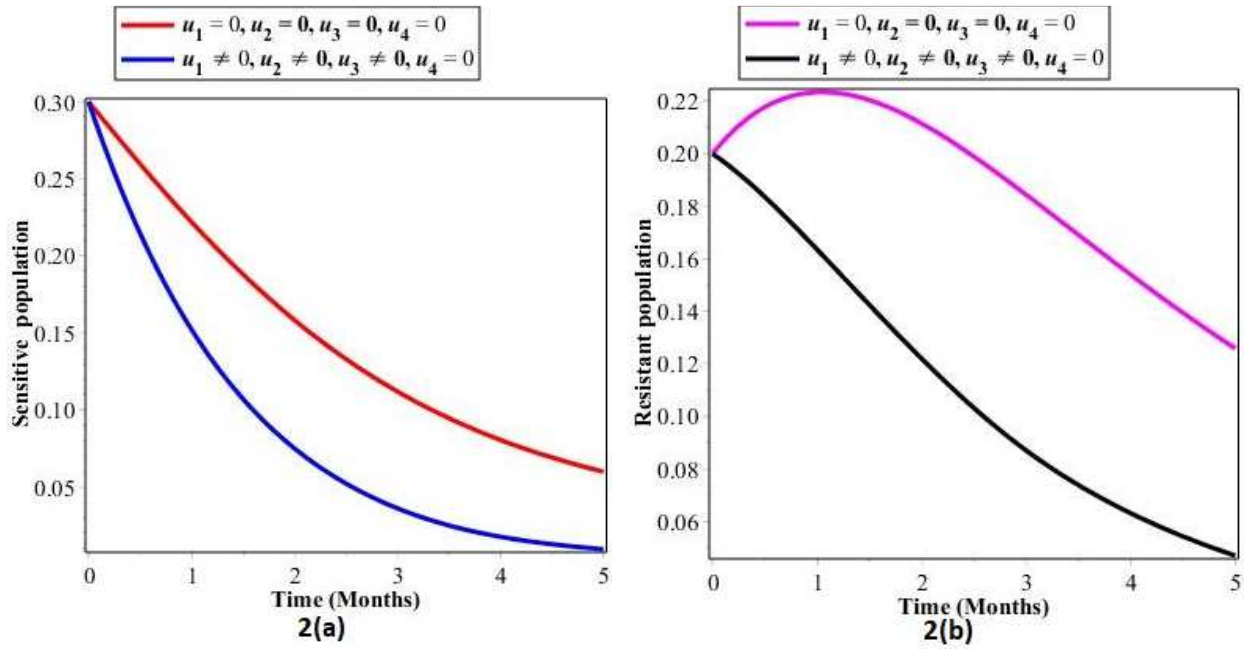

Figure 2. *Graph displaying the impact of intervention strategies:* $u_1 \neq 0, u_2 \neq 0$ *and* $u_3 \neq 0$.

Figures 2(a) and 2(b) shows the case when application of the control efforts $u_1 \neq 0, u_2 \neq 0$ and $u_3 \neq 0$ are non-zero. Prevention technique, u_1 and treating efforts u_2 and u_3 are applied on the function J and equating immunity intervention u_4 to zero. Figures 2(a) and 2(b) confirms that implementing this approach reduces the drug sensitive infective individuals (i) and drug resistant infective individuals (r_T) significantly in the community. This technique showed that the meningitis infective population gave a significant positive change since prevention efforts like education campaigns and treatment interventions were employed against meningitis.

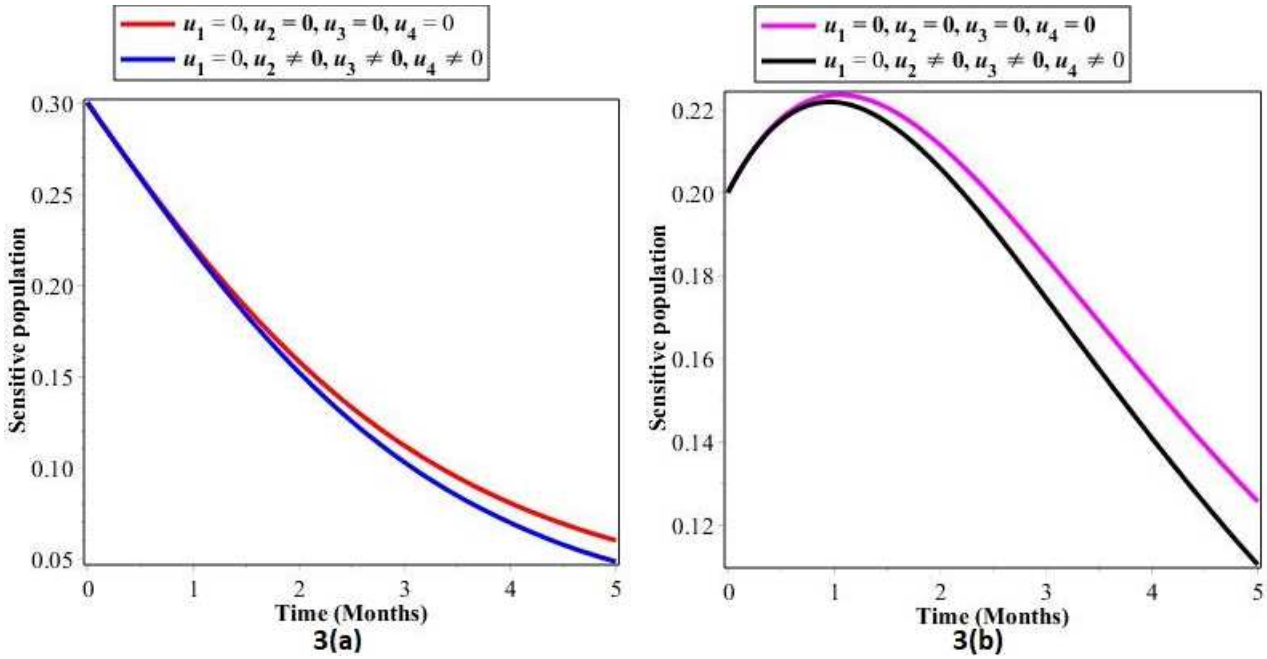

Figure 3. *Graph displaying the impact of intervention strategies:* $u_2 \neq 0, u_3 \neq 0$ *and* $u_4 \neq 0$.

Figure 3(a) and 3(b) shows the impact of non-zero controls $u_2 \neq 0, u_3 \neq 0$ and $u_4 \neq 0$ with the prevention intervention u_1 equal to zero. Figure 4(a) and 4(b) shows the effect of optimal treatment controls only. Optimizing the value of J over u_2 and u_3 with u_4 and u_1 equated to zero. Notably, figures 3(a), 3(b) and figure 4(a), 4(b) shows that due to the control strategies, the number of drug sensitive infective individuals (i) and drug resistant infective individuals (r_T) displays little effect on decreasing the number of drug sensitive individuals (i),

while there is little notable effect of the controls on timing and attainment of peak infections by drug resistant individuals (r_T). However, the controls result to infections decrease in the community.

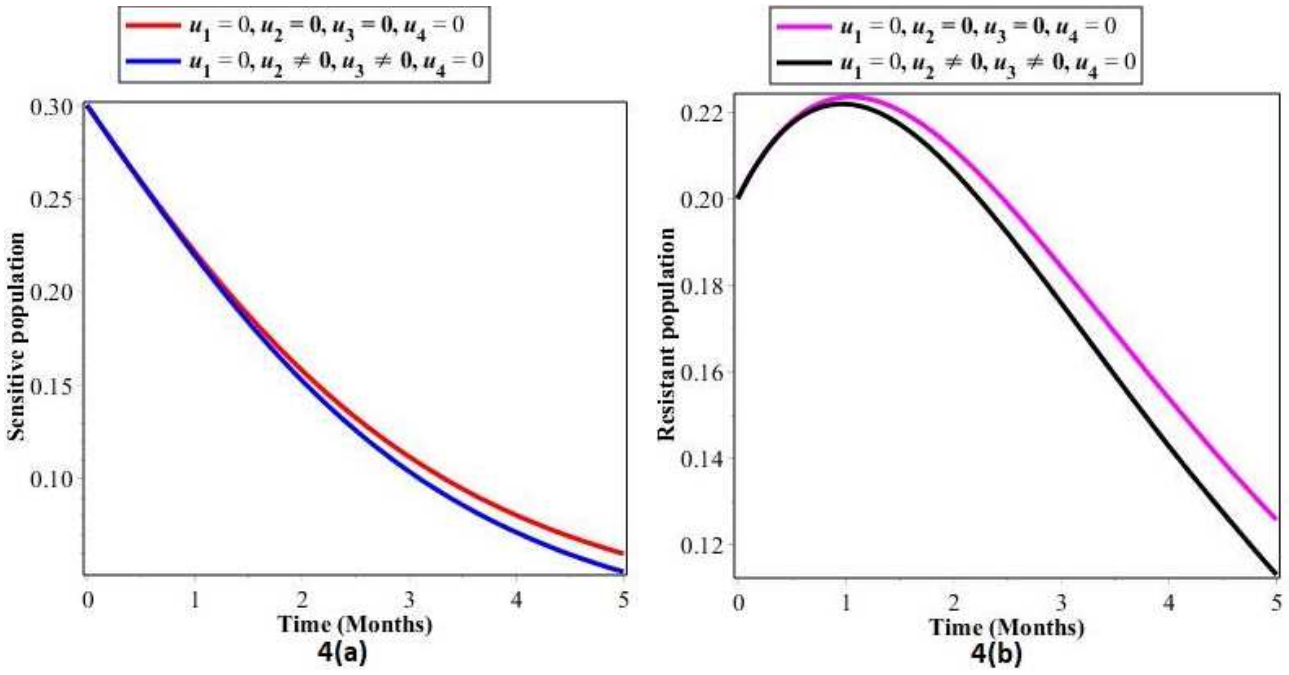

Figure 4. *Graph displaying the impact of intervention strategies:* $u_2 \neq 0$ *and* $u_3 \neq 0$.

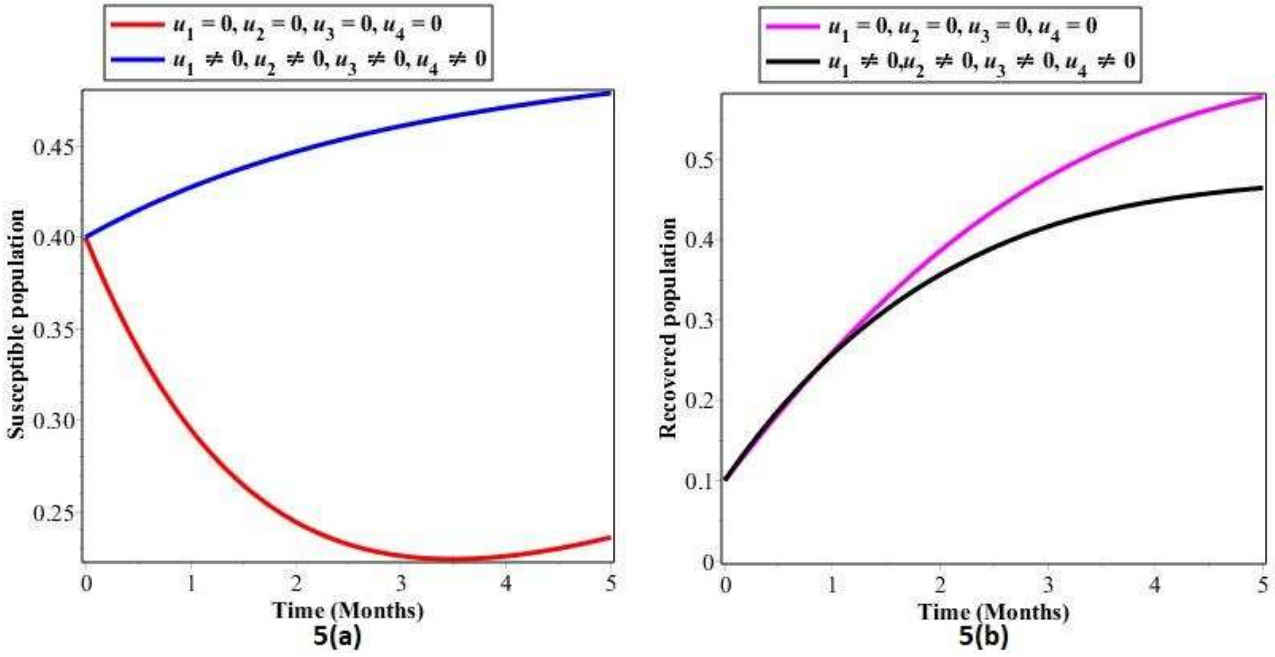

Figure 5. *Graph displaying the impact of intervention strategies: susceptible and recovered population.*

Figures 5(a), 5(b) and 6(a), 6(b) shows the effect of employing all the control efforts. Using no-zero controls u_1, u_2, u_3 and u_4 to maximize J. Figures 6(a) and 6(b) confirms the significant contribution of all controls, the drug sensitive infective persons (i) and drug resistant infective persons (r_T) showed a remarkable sharp decrease in the community. While at the same time Figures 5(a) and 5(b) displayed a gradual increase of both susceptible and recoveries.

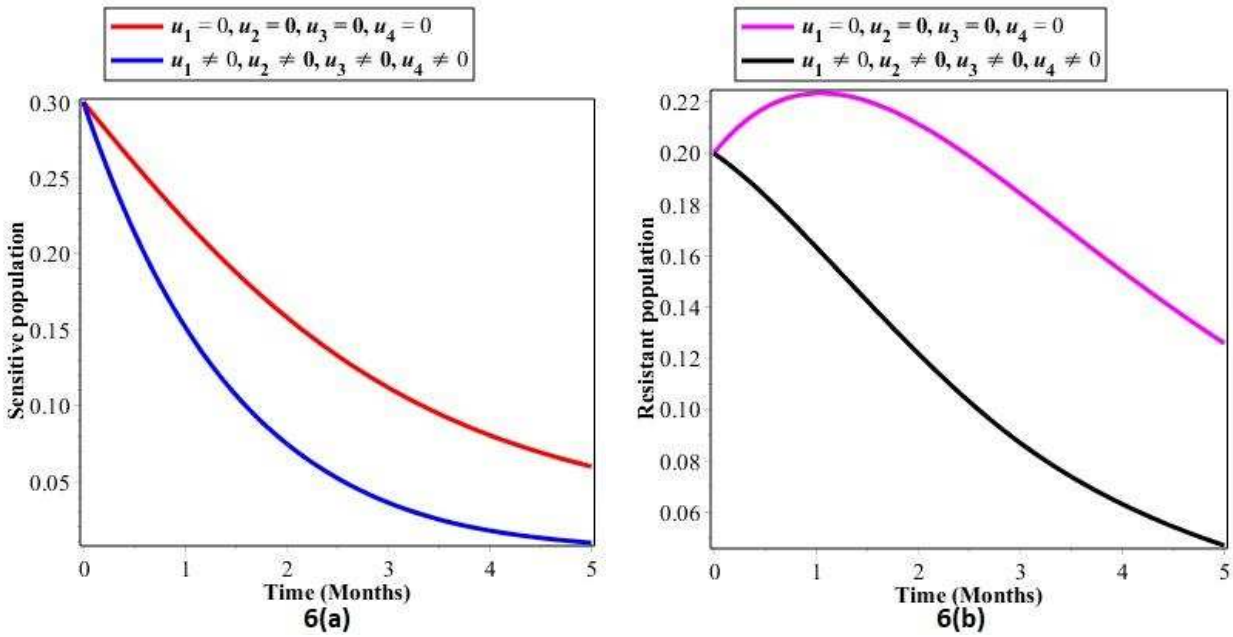

Figure 6. *Graph displaying the impact of all intervention strategies:* $u_1 \neq 0, u_2 \neq 0, u_3 \neq 0$ *and* $u_4 \neq 0$.

7. Conclusions

The article formulates and analyzes an epidemic mathematical model that predicts the dynamics of meningococcal meningitis infection that maximizes prevention efforts, treatments, immunity control and also performed qualitative and quantitative study of the optimality system. The meningitis system was significantly studied to reveal key parameters that shape its transmission in the community. The meningitis only model has a locally-stable zero disease state when the effective reproduction value $R_e < 1$. The model has a unique persistent state whenever $R_e > 1$. The study reveals that infection rate and vaccination proportion are the most sensitive parameter values to be targeted to eradicate the meningitis disease. Focusing on meningitis employed techniques might result to wipe out of meningitis as clearly shown in Figure 2(a) and (b), where the population of meningitis infective persons are observed to be decreasing sharply to attain disease free state. Notably, application of all controls has demonstrated to be effective in eliminating infections from the community. Thus, there is need for further studies to determine the approach that is cost-effective technique in the case of less resources as witnessed in many developing economies.

Conflict of Interest

The authors declare no conflict of interests.

Data Availability

The meningitis model data were obtained from published articles and reported studies which have been cited accordingly. Some of the parameter values are estimated.

References

[1] Afolabi, M. A., Adewoye, K. S., Folorunso, A. I., & Omoloye, M. A. (2021). A mathematical model on transmission dynamics of meningococcal meningitis. Iconic Research and Engineering Journal, (pp. 59-66).

[2] Agusto, F. & Leite, M. (2019). Optimal control and cost-effective analysis of the 2017 meningitis outbreak in nigeria. Infectious Disease Modelling, 4, 161-187.

[3] Asamoah, J. K. K., Nyabadza, F., Seidu, B., Chand, M., & Dutta, H. (2018). Mathematical modelling of bacterial meningitis transmission dynamics with control measures. Computational and mathematical methods in medicine, 2018.

[4] Blyuss, K. B. (2016). Mathematical modelling of the dynamics of meningococcal meningitis in africa. In UK Success Stories in Industrial Mathematics (pp. 221-226). Springer.

[5] Chitnis, N., Hyman, J. M., & Cushing, J. M. (2008). Determining important parameters in the spread of malaria through the sensitivity analysis of a mathematical model. Bulletin of mathematical biology, 70 (5), 1272-1296.

[6] Fleming, W. H. & Rishel, R. W. (2012). Deterministic and stochastic optimal control, volume 1. Springer Science & Business Media.

[7] Grassly, N. C. & Fraser, C. (2008). Mathematical models of infectious disease transmission. Nature Reviews Microbiology, 6 (6), 477-487.

[8] Irving, T., Blyuss, K., Colijn, C., & Trotter, C. (2012). Modelling meningococcal meningitis in the african meningitis belt. Epidemiology and Infection, 140 (5), 897-905.

[9] Karachaliou, A., Conlan, A. J., Preziosi, M.-P., & Trotter, C. L. (2015). Modeling long-term vaccination strategies with menafrivac in the african meningitis belt. Clinical Infectious Diseases, 61 (suppl_5), S594-S600.

[10] Mbaeyi, S. A., Bozio, C. H., Duffy, J., Rubin, L. G., Hariri, S., Stephens, D. S., & MacNeil, J. R. (2020). Meningococcal vaccination: recommendations of the advisory committee on immunization practices, united states, 2020. MMWR Recommendations and Reports, 69 (9), 1.

[11] Mbaeyi Sarah, Jonathan Duffy, M. L. A. (2020). Meningococcal disease. CDC Reports.

[12] mondiale de la Santé, O., Organization, W. H., et al. (2015). Meningococcal disease control in countries of the african meningitis belt, 2014. Weekly Epidemiological Record= Relevé épidémiologique hebdomadaire, 90 (13), 123-131.

[13] Organization, W. H. et al. (2007). Standardized treatment of bacterial meningitis in Africa in epidemic and non-epidemic situations. Technical report, World Health Organization.

[14] Organization, W. H. et al. (2014). Meningococcal disease control in countries of the african meningitis belt, 2013. Weekly Epidemiological Record= Relevé épidémiologique hebdomadaire, 89 (20), 206–214.

[15] Pizza, M., Bekkat-Berkani, R., & Rappuoli, R. (2020). Vaccines against meningococcal diseases. Microorganisms, 8 (10), 1521.

[16] Pontryagin, L. S. (2018). Mathematical theory of optimal processes. Routledge.

[17] Tilahun, G. T. (2019a). Modeling co-dynamics of pneumonia and meningitis diseases. Advances in Difference Equations, 2019 (1), 1-18.

[18] Tilahun, G. T. (2019b). Optimal control analysis of pneumonia and meningitis coinfection. Computational and Mathematical Methods in Medicine, 2019.

[19] Vereen, K. (2008). An scir model of meningococcal meningitis.

[20] Workineh, Y. H. & Kassa, S. M. (2021). Optimal control of the spread of meningitis: in the presence of behaviour change of the society and information dependent vaccination. Commun. Math. Biol. Neurosci., 2021, Article-ID.

[21] Yano, K. T. (2018). SEIR Childhood Disease Model with Constant Vaccination Strategy. PhD thesis, JKUAT-PAUSTI.

[22] Yusuf, T. T. & Olayinka, A. O. (1979). Optimal control of meningococcal meningitis transmission dynamics: A case study of Nigeria.

[23] Jacobsson, S. (2016). Despite successful vaccines Neisseria meningitidis strikes again. *The Lancet. Infectious Diseases*, *16* (11), 1212-1213.