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How to Deduce the Remaining 23 Valid Syllogisms from the Validity of the Syllogism *EIO-1*

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Abstract: Syllogistic reasoning plays an important role in human reasoning, and has been widely studied from Aristotle onward. In previous studies, when deriving all the other valid syllogisms, at least two valid syllogisms were taken as the basic axioms. While this paper derives all other valid syllogisms only from one valid syllogism. On the basis of generalized quantifier theory and set theory, this paper shows that the remaining 23 valid syllogisms can be derived only from the syllogism *EIO-1* by making the best of the definitions of three negative quantifiers of Aristotelian quantifiers, the symmetry of Aristotelian quantifiers *no* and *some*, and several propositional reasoning rules such as anti-syllogism rules and the subsequent weakening rule, and so on. This paper syntactically provides a simple and reasonable mathematical model for studying other kinds of syllogisms, such as generalized syllogistic, rational syllogistic, Aristotelian modal syllogistic and generalized modal syllogistic. And this research shows that formalized logic has the characteristics of structuralism, that is, it studies not only the forms and laws of thinking, but also the structure of thinking objects and the relationship between structures. It is hoped that this formal and innovative research is not only beneficial to the further development of various syllogistic logics, but also to natural language information processing in computer science, and also to knowledge representation and knowledge reasoning in Artificial Intelligence.

Keywords: Generalized Quantifier Theory, Aristotelian Syllogisms, Axioms, Aristotelian Quantifiers, Rules

1. Introduction

Syllogistic reasoning is a common form of reasoning in natural language [1-3], which has been widely studied and plays an important role in logic [4-10]. Aristotelian syllogistic logic mainly studies the semantic properties and reasoning properties of four Aristotelian quantifiers, that is, *all*, *no*, *some* and *not all* [11, 12]. The major premise, minor premise and conclusion of an Aristotelian syllogism can be composed of the four sorts of propositions *A*, *E*, *I* and *O*, and the middle term has four different positions in the major and minor premises, hence there are (4×4×4×4=) 256 kinds of Aristotelian syllogisms, and only 24 syllogisms are valid among them [13, 14].

Łukasiewicz [15] formally derived the other 22 valid Aristotelian syllogisms from the two Aristotelian syllogisms *AAA-1* and *AII-3* by using propositional reasoning rules. On the basis of Łukasiewicz's work, and by means of the knowledge of first-order logic, Shushan Cai [16] axiomatized Aristotelian syllogistic logic from the two syllogisms *AAA-1* and *AII-3* and the fact $aEb \rightarrow bEa$ (that is, the symmetry of Aristotelian quantifier *no*). By means of generalized quantifier theory, Xiaojun Zhang and Sheng Li [13] formally proved the remaining 22 valid Aristotelian syllogisms from the two Aristotelian syllogisms *AAA-1* and *EAE-1*. Mengyao Huang and Xiaojun Zhang [17] expounded the work of Łukasiewicz [15] by making use of generalized quantifier theory. Unless otherwise specified, all of syllogisms in the following are Aristotelian syllogisms.

There are many results of studying Aristotelian syllogistic logic by different methods, such as Westerståhl [12], Moss [3, 8], Endrullis and Moss [5], and so on. As far as we know, there are at least two syllogisms as the reasoning basis when one tries to deduce the remaining valid syllogisms [18]. While this paper takes only one syllogism (that is, *EIO-1*) as the reasoning basis in order to deduce the remaining 23 valid syllogisms. More specifically, this paper syntactically proves that the remaining 23 valid syllogisms can be derived only based on the valid syllogism *EIO-1* by making full use of generalized quantifier theory and set theory.

2. The Structure of Syllogisms and Their Formalization

A syllogism is composed of three categorical propositions. Every categorical proposition has the form: *Q(x, y)*, in which *Q* represents any of the four Aristotelian quantifiers (that is, *all*, *no*, *some* and *not all*), and *x* is the subject variable, *y* is the predicate variable.

(1) *all(x, y)* means "all *x*s are *y*", which is an universal affirmative proposition, and called the proposition *A*.

(2) *no(x, y)* means "no *x*s are *y*", that is, "all *x*s are not *y*", which is an universal negative proposition, and called the proposition *E*.

(3) *some(x, y)* means "some *x*s are *y*", which is a particular affirmative proposition, and called the proposition *I*.

(4) *not all(x, y)* means "not all *x*s are *y*", that is, "some *x*s are not *y*", which is a particular negative proposition, and called the proposition *O*.

The figures of syllogisms are determined by the position of the middle term, and its definition is as usual. For example, "*no ys* are *z*, and *some x*s are *y*, then *not all x*s are *z*", in which *x*, *y* and *z* represent the lexical variables in the syllogism. This syllogism is the first figure and is composed of the categorical proposition *E*, *I*, and *O,* respectively. Therefore, it is the first figures syllogism *EIO*, which can be denoted as *EIO-1*. The syllogism can be formalized as *no(x, y)→(some(x, y)→not all(x, y))*. The formalization of other syllogisms is similar to this.

3. Syntax and Semantics of Aristotelian Syllogism Logic

The initial symbols, formation rules and related definitions of Aristotelian syllogistic logic are given respectively in the following.

3.1. Primitive Symbols

(1) lexical variables: x, y, *z*
(2) unary negative operator: ¬
(3) binary implication operator: →
(4) quantifier: *all*
(5) brackets: (,)

3.2. Formation Rules

(1) If *Q* is a quantifier, *x* and *y* are lexical variables, then *Q(x, y)* is a well-formed formula.

(2) If *p* and *q* are well-formed formulas, then *p→q* are well-formed formulas.

(3) Only the formulas obtained through (1) and (2) are well-formed formulas.

For example, *all(x, y)*, and *all(x, y)→¬all(y, z)* are well-formed formulas, which read respectively as '*all x*s are *y*', and 'if *all x*s are *y*, then that *all y*s are *z* is false'. Others are similar.

Let *D* be the domain of lexical variables, and *Q* be a quantifier, then the outer quantifier of *Q* is denoted as ¬*Q*, the inner quantifier of *Q* is denoted as *Q*¬, and the dual quantifier of *Q* is denoted as ¬*Q*¬. For example: *not all*=¬*all*, *no*=*all*¬, *some*=¬*all*¬, so the quantifier used as the initial symbol in this paper is only the Aristotelian quantifier *all*, and the other three Aristotelian quantifiers can be obtained by the definition of negative quantifiers.

3.3. Related Definitions

(1) Definition of connective ∧:
$(p\wedge q) =_{\text{def}} \neg(p\rightarrow\neg q)$;

(2) Definition of connective ↔:
$(p\leftrightarrow q) =_{\text{def}} ((p\rightarrow q)\wedge(q\rightarrow p))$;

(3) Definition of inner negative quantifier:
$Q\neg(x, y) =_{\text{def}} Q(x, D-y)$;

(4) Definition of outer negative quantifier:
$\neg Q(x, y) =_{\text{def}}$ It is not that *Q(x, y)*;

(5) Definition of the quantifier *not all*:
not all(x, y) $=_{\text{def}}$ ¬*all(x, y)*;

(6) Definition of the quantifier *no*:
no(x, y) $=_{\text{def}}$ *all*¬*(x, y)*;

(7) Definition of the quantifier *some*:
some(x, y) $=_{\text{def}}$ ¬*all*¬*(x, y)*.

This paper only studies the propositions containing *all(x, y)*, *no(x, y)*, *some(x, y)* and *not all(x, y)*, so there is no recursion of any kind.

4. Axiom System of Aristotelian Syllogism Logic

In the following, ⊢ represents a proposition or syllogism that can be proved. For example, the syllogism EIO-1 can be proved, and denoted as ⊢*no(x, y)→(some(x, y)→not all(x, y))*. The other notations are similar. The verifiable Aristotelian syllogisms in the system can be derived from the following basic axioms and reasoning rules.

4.1. Basic Axioms

(1) A0: if α is a valid formula in propositional logic, then ⊢α.

(2) A1: ⊢*all(x, x)*.

(3) A2: ⊢*some(x, x)*.

(4) A3 (that is, the syllogism *EIO-1*):
⊢*no(y, z)→(some(x, y)→not all(x, z))*.

The following reasoning rules in propositional logic will be used later.

4.2. Reasoning Rules

Aristotelian syllogistic logic is an extension of classical propositional logic, so the following reasoning rules in the latter are also applicable in the former. In the following rules, α, β, γ and δ are well-formed formulas.

(1) Rule 1 (Replacement rule): if the formula α is obtained from the formula β by means of "replacing one variable with another", then $\vdash\alpha$ can be derived from $\vdash\beta$.
(2) Rule 2 (Modus Ponens): $\vdash\beta$ can be derived from $\vdash(\alpha\rightarrow\beta)$ and $\vdash\alpha$.
(3) Rule 3 (Definiens and definendum interchange): $\vdash(...\beta...)$ can be obtained from $\vdash(...\alpha...)$ and $\alpha=_{\text{def}}\beta$, and vice versa.
(4) Rule 4 (Substitution of equivalents): $\vdash(...\beta...)$ can be derived from from $\vdash(...\alpha...)$ and $\alpha\leftrightarrow\beta$, and vice versa.
(5) Rule 5 (Double negative): $\vdash\alpha$ can be derived from $\vdash\neg\neg\alpha$, and vice versa.
(6) Rule 6 (Antecedent interchange): $\vdash(\beta\rightarrow(\alpha\rightarrow\gamma))$ can be obtained from $\vdash(\alpha\rightarrow(\beta\rightarrow\gamma))$.
(7) Rule 7 (Subsequent weakening): $\vdash(\alpha\rightarrow(\beta\rightarrow\delta))$ can be derived from $\vdash(\alpha\rightarrow(\beta\rightarrow\gamma))$ and $\vdash(\gamma\rightarrow\delta)$.
(8) Rule 8 (Reverse rule): From $\vdash(\alpha\rightarrow\beta)$ infer $\vdash(\neg\beta\rightarrow\neg\alpha)$.
(9) Rule 9 (Rule *A* of anti-syllogism): From $\vdash(\alpha\rightarrow(\beta\rightarrow\gamma))$ infer $\vdash(\alpha\rightarrow(\neg\gamma\rightarrow\neg\beta))$.
(10) Rule 10 (Rule *B* of anti-Syllogism): From $\vdash(\alpha\rightarrow(\beta\rightarrow\gamma))$ infer $\vdash(\beta\rightarrow(\neg\gamma\rightarrow\neg\alpha))$.

4.3. Relevant Facts

According to generalized quantifier theory [13, 19], among the four Aristotelian quantifiers (that is, *all*, *no*, *some* and *not all*), any three Aristotelian quantifiers are one of the three kinds of negative (that is, inner negative, outer negative and dual negative) quantifiers of the other Aristotelian quantifier. Specifically, (1) *all* and *no*, *some* and *not all* are inner negations each other, that is, *all*=*no*¬, *no*=*all*¬; *some*=*not all*¬; *not all*=*some*¬ (i.e. the following fact 1); (2) *all* and *not all*, *some* and *no* are outer negative each other, that is, *all*=¬*not all*, *not all*=¬*all*; *some*=¬*no*, *no*= ¬*some* (i.e. the following fact 2).

On the basis of the above definitions, reasoning rules, and axioms, the following facts can be easily proved by means of the above basic axioms and reasoning rules. And these facts are the definitions or facts in generalized quantifier theory [13, 19], thus their detailed proofs are omitted here.

Fact 1 (inner negation):
(1) ⊢*all(x, y)*↔*no*¬*(x, y)*;
(2) ⊢*no(x, y)*↔*all*¬*(x, y)*;
(3) ⊢*some(x, y)*↔*not all*¬*(x, y)*;
(4) ⊢*not all(x, y)*↔*some*¬*(x, y)*.

Fact 2 (outer negation):
(1) ⊢*all(x, y)*↔¬*not all(x, y)*;
(2) ⊢*not all(x, y)*↔¬*all(x, y)*;
(3) ⊢*some(x, y)*↔¬*no(x, y)*;
(4) ⊢*no(x, y)*↔¬*some(x, y)*.

Fact 3 (symmetry of *some* and *no*):
(1) (symmetry of *some*): ⊢*some(x, y)*↔*some(y, x)*;
(2) (symmetry of *no*): ⊢*no(x, y)*↔*no(y, x)*.

Fact 4 (assertoric subalternations):
(1) ⊢*no(x, y)*→*not all(x, y)*;
(2) ⊢*all(x, y)*→*some(x, y)*.

4.4. Reducible Relations Between / Among Syllogisms

In the following theorem 1, *EIO-1*⇒*EIO-2* means that the validity of syllogism *EIO-2* can be obtained from the validity of syllogism *EIO-1*. In other words, there is a reducible relationship between the two Aristotelian syllogisms. Others are similar. In fact, the syllogism *EIO-1* is the basic axiom A3.

Theorem 1: The remaining 23 valid syllogisms can be derived only from the syllogism *EIO-1*. According to the order of proof, we find the following reducible relations between/among syllogisms:

(1) *EIO-1*⇒*EIO-2*
(2) *EIO-1*⇒*EIO-3*
(3) *EIO-1*⇒*EIO-3*⇒*EIO-4*
(4) *EIO-1*⇒*EAE-2*
(5) *EIO-1*⇒*EAE-2*⇒*EAE-1*
(6) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AEE-4*
(7) *EIO-1*⇒*EAE-2*⇒*AEE-2*
(8) *EIO-1*⇒*AII-3*
(9) *EIO-1*⇒*AII-3*⇒*AII-1*
(10) *EIO-1*⇒*AII-3*⇒*AII-1*⇒*IAI-4*
(11) *EIO-1*⇒*AII-3*⇒*IAI-3*
(12) *EIO-1*⇒*EAE-2*⇒*EAO-2*
(13) *EIO-1*⇒*EAE-2*⇒*EAO-2*⇒*EAO-1*
(14) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AEE-4*⇒*AEO-4*
(15) *EIO-1*⇒*AEE-2*⇒*AEO-2*
(16) *EIO-1*⇒*EAE-2*⇒*EAO-2*⇒*AAI-3*
(17) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AAA-1*
(18) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AAA-1*⇒*AAI-1*
(19) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AAA-1*⇒*AAI-1*⇒*AAI-4*
(20) *EIO-1*⇒*EIO-2*⇒*AOO-2*
(21) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AAI-1*⇒*EAO-4*
(22) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AAI-1*⇒*EAO-4*⇒*EAO-3*
(23) *EIO-1*⇒*EAE-2*⇒*EAE-1*⇒*AAA-1*⇒*OAO-3*

Proof:

[1] ⊢no(y, z)→(some(x, y)→not all(x, z))	*(i. e. EIO-1, that is basic axiom A3)*
[2] ⊢no(y, z)↔no(z, y)	*(by Fact 3 and Rule 1)*
[3] ⊢no(z, y)→(some(x, y)→not all(x, z))	*(i.e. EIO-2, by [1, 2] and Rule 4)*
[4] ⊢some(x, y)↔some(y, x)	*(by Fact 3 and Rule 1)*
[5] ⊢no(y, z)→(some(y, x)→not all(x, z))	*(i.e. EIO-3, by [1, 4] and Rule 4)*
[6] ⊢no(z, y)→(some(y, x)→not all(x, z))	*(i.e. EIO-4, by [2, 5] and Rule 4)*

[7] ⊢no(y, z)→(¬not all(x, z)→¬some(x, y)) *(by [1] and Rule 9)*
[8] ⊢no(y, z)→(all(x, z)→no(x, y)) *(i.e. EAE-2, by [7], Fact 2 and Rule 4)*
[9] ⊢no(z, y)→(all(x, z)→no(x, y)) *(i.e. EAE-1, by [2, 8] and Rule 4)*
[10] ⊢no(x, y)↔no(y, x) *(by Fact 3 and Rule 1)*
[11] ⊢all(x, z)→(no(z, y)→no(y, x)) *(i.e. AEE-4, by [9, 10] and Rule 6)*
[12] ⊢no(y, z)→(all(x, z)→no(y, x)) *(by [8, 10] and Rule 4)*
[13] ⊢all(x, z)→(no(y, z)→no(y, x)) *(i.e. AEE-2, by [12] and Rule 6)*
[14] ⊢some(x, y)→(¬not all(x, z)→¬no(y, z)) *(by [1] and Rule 10)*
[15] ⊢some(x, y)→(all(x, z)→some(y, z)) *(by [14], Fact 2 and Rule 4)*
[16] ⊢all(x, z)→(some(x, y)→some(y, z)) *(i.e. AII-3, by [15] and Rule 6)*
[17] ⊢all(x, z)→(some(y, x)→some(y, z)) *(i.e. AII-1, by [4, 16] and Rule 4)*
[18] ⊢some(y, z)↔some(z, y) *(by Fact 3 and Rule 1)*
[19] ⊢all(x, z)→(some(y, x)→some(z, y)) *(by [17, 18] and Rule 4)*
[20] ⊢some(y, x)→(all(x, z)→some(z, y)) *(i.e. IAI-4, by [19] and Rule 6)*
[21] ⊢all(x, z)→(some(x, y)→some(z, y)) *(by [4, 16] and Rule 4)*
[22] ⊢some(x, y)→(all(x, z)→some(z, y)) *(i.e. IAI-3, by [21] and Rule 6)*
[23] ⊢no(x, y)→not all(x, y) *(by Fact 4 and Rule 1)*
[24] ⊢no(y, z)→(all(x, z)→not all(x, y)) *(i.e. EAO-2, by [8, 23] and Rule 7)*
[25] ⊢all(x, z)→(no(z, y)→not all(x, y)) *(by [2, 24] and Rule 4)*
[26] ⊢no(z, y)→(all(x, z)→not all(x, y)) *(i.e. EAO-1, by [25] and Rule 6)*
[27] ⊢no(y, x)→not all(y, x) *(by Fact 4 and Rule 1)*
[28] ⊢all(x, z)→(no(z, y)→not all(y, x)) *(i.e. AEO-4, by [11, 27] and Rule 7)*
[29] ⊢all(x, z)→(no(y, z)→not all(y, x)) *(i.e. AEO-2, by [13, 27] and Rule 7)*
[30] ⊢all(x, z)→(¬not all(x, y)→¬no(y, z)) *(by [24] and Rule 9)*
[31] ⊢all(x, z)→(all(x, y)→some(y, z)) *(i.e. AAI-3, by [30], Fact 2 and Rule 4)*
[32] ⊢all¬(z, y)→(all(x, z)→all¬(x, y)) *(by [9] and the definition of no)*
[33] ⊢all(z, D¬y)→(all(x, z)→all(x, D¬y)) *(by [32] and the definition of inner negative quantifier)*
[34] ⊢all(z, y)→(all(x, z)→all(x, y)) *(i.e. AAA-1, by [33] and Rule 1)*
[35] ⊢all(x, y)→some(x, y) *(by Fact 4 and Rule 1)*
[36] ⊢all(z, y)→(all(x, z)→some(x, y)) *(i.e. AAI-1, by [34, 35] and Rule 1)*
[37] ⊢all(z, y)→(all(x, z)→some(y, x)) *(by [4, 36] and Rule 1)*
[38] ⊢all(x, z)→(all(z, y)→some(y, x)) *(i.e. AAI-4, by [37] and Rule 6)*
[39] ⊢all¬(z, y)→(not all¬(x, y)→not all(x, z)) *(by [3] and the definition of no and some)*
[40] ⊢all(z, D¬y)→(not all(x, D¬y)→not all(x, z)) *(by [39] and the definition of inner negative quantifier)*
[41] ⊢all(z, y)→(not all(x, y)→not all(x, z)) *(i.e. AOO-2, by [40] and Rule 1)*
[42] ⊢all(x, z)→(¬some(y, x)→¬all(z, y)) *(by [38] and Rule 9)*
[43] ⊢all(x, z)→(no(y, x)→not all(z, y)) *(by [42], Fact 2 and Rule 4)*
[44] ⊢no(y, x)→(all(x, z)→not all(z, y)) *(i.e. EAO-4, by [43] and Rule 6)*
[45] ⊢no(x, y)→(all(x, z)→not all(z, y)) *(i.e. EAO-3, by [10, 44] and Rule 4)*
[46] ⊢all(x, z)→(¬all(x, y)→¬all(z, y)) *(by [34] and Rule 10)*
[47] ⊢all(x, z)→(not all(x, y)→not all(z, y)) *(by [46], Fact 2 and Rule 4)*
[48] ⊢not all(x, y)→(all(x, z)→not all(z, y)) *(i.e. OAO-3, by [47] and Rule 6)*

It can be seen from theorem 1 that the remaining 23 valid syllogisms can be derived only from the valid syllogism *EIO-1* through 48 steps by making full of generalized quantifier theory and the reasoning rules in propositional logic.

Moss [8], Beihai Zhou et al. [20], and Xiaojun Zhang [21] have studied the soundness and completeness of Aristotelian syllogistic logic, however, these studies need to be refined and perfected. For example, Beihai Zhou et al. [20], took four axioms (that is, *all(x, x)*, *all(x, ¬¬x), all(¬¬x, x)* and *no(x, ¬x)*) as the basic axioms, and took the two syllogisms *AAA-1* and *EAE-1* as the initial rules. Using the method of canonical model, they proved the soundness and completeness of Aristotelian syllogism logic. But this proof was complex and lengthy.

While this paper only takes *all(x, x)*, *some(x, x)* and the syllogism *EIO-1* as basic axioms, and uses the reasoning rules in propositional logic and generalized quantifier theory to establish a minimalist formal axiom system for Aristotelian syllogism logic. Then can we simplify the proof of the soundness and completeness of Aristotelian syllogism logic by using generalized quantifier theory? This problem needs further study.

5. Conclusion

This paper shows that the remaining 23 valid syllogisms can be derived only from the syllogisms *EIO-1* by making the best of the definitions of three negative quantifiers of Aristotelian quantifiers in generalized quantifier theory, the symmetry of Aristotelian quantifiers *no* and *some*, and reasoning rules in propositional logic. And the proof of reduction between/among different figures and forms of syllogisms is simple and clear.

This innovative research shows that formalized logic has the characteristics of structuralism, that is, it studies not only the forms and laws of thinking, but also the structure of thinking objects and the relationship between structures. And this paper provides a research paradigm for other kinds of syllogistic, such as generalized syllogistic, rational syllogistic, Aristotelian modal syllogistic and generalized modal syllogistic. Therefore, this study is not only beneficial to the further development of various syllogistic logics, but also to natural language information processing in computer science, and also to knowledge representation and knowledge reasoning in Artificial Intelligence.

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References

[1] N. Chater and M. Oaksford, the probability Heuristics model of syllogistic reasoning, *Cognitive Psychology*, Vol. 38, 1999, pp. 191-258.

[2] G. Patzig, *Aristotle's Theory of the Syllogism*, J. Barnes (trans.), Dordrecht: D. Reidel, 1969.

[3] L. S. Moss, Syllogistic logics with verbs, *Journal of Logic and Computation*, Vol. 20, No. 4, 2010, pp. 947-967.

[4] P. Murinová P and V. Novák, A formal theory of generalized intermediate syllogisms, *Fuzzy Sets and Systems*, Vol. 186, 2012, pp. 47-80.

[5] J. Endrullis, and L. S. Moss, Syllogistic logic with 'Most', in V. de Paiva et al. (eds.), *Logic, Language, Information, and Computation,* 2015, pp. 124-139.

[6] J. Łukasiewicz, *Aristotle's Syllogistic from the Standpoint of Modern Formal Logic*, Clarendon Press, Oxford, 1951.

[7] J. N. Martin, Aristotle's natural deduction reconsidered, *History and Philosophy of Logic*, Vol. 18, No. 1 1997, pp. 1-15.

[8] L. S. Moss, Completeness theorems for syllogistic fragments, in F. Hamm and S. Kepser (eds.), *Logics* for *Linguistic Structures*, Mouton de Gruyter, Berlin, 2008, pp. 143–173.

[9] J. van Benthem, Questions about quantifiers, *Journal of Symbol Logic*, Vol. 49, No. 2, 1984, pp. 443-466.

[10] D. Westerståhl, Aristotelian syllogisms and generalized quantifiers, *Studia Logica*, Vol. XLVII, No. 4, 1989, pp. 577-585.

[11] Xiaojun Zhang, *A Study of Generalized Quantifier Theory*, Xiamen University Press, 2014. (in Chinese).

[12] D. Westerståhl, Quantifiers in formal and natural languages, in D. M. Gabbay and F. Guenthner (eds.), *Handbook of Philosophical Logic*, Vol. 14, 2007, pp. 227-242.

[13] Xiaojun Zhang, Sheng Li. Research on the formalization and axiomatization of traditional syllogisms, *Journal of Hubei University (Philosophy and Social Sciences)*, No. 6, 2016, pp. 32-37. (in Chinese).

[14] Yijiang Hao. Formal research on discourse reasoning in natural language. *Journal of Hunan University of Science and Technology (Social Sciences Edition)*, 2016 (1): 33-37. (in Chinese).

[15] Łukasiewicz J. *Aristotle's Syllogistic: From the Standpoint of Modern Formal Logic*. Second edition, Oxford: Clerndon Press, 1957.

[16] Shushan Cai. A formal system of Aristotle's syllogism different from that of Łukasiewicz. *Philosophical research*, 1988 (4): 33- 41. (in Chinese).

[17] Mengyao Huang, Xiaojun Zhang. Assertion or rejection of Łukasiewicz's assertoric syllogism system ŁA. *Journal of Chongqing University of Science and Technology (Social Sciences Edition)*, 2020 (2): 10-18. (in Chinese).

[18] Xiaojun Zhang. Axiomatization of Aristotelian syllogistic logic based on generalized quantifier theory. Applied and Computational Mathematics. Vol. 7, No. 3, 2018, pp. 167-172.

[19] S. Peters, and D. Westerståhl, *Quantifiers in Language and Logic*, Claredon Press, Oxford, 2006.

[20] Beihai Zhou, Qiang Wang, Zhi Zheng. Aristotle's division lattice and Aristotelian logic. *Logic research*, 2018 (2): 2-20. (in Chinese).

[21] Xiaojun Zhang. *Research on Extended Syllogism for Natural Language Information Processing*. Beijing: Science Press, 2020. (in Chinese).